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ELECTROMAGNETIC AND TEMPERATURE FIELDS IN DIRECT RESISTANCE HEATING OF COMPOUND AXISYMMETRIC BODIES

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A nonstationary coupled problem of thermoelectrodynamics is formulated for resistance heating of dissimilar ferro- and paramagnetic bodies by an alternating current. An iterative algorithm for solving this problem by finite-difference methods is proposed. Temperature and electromagnetic-field distributions are obtained for the processes of direct resistance heating both in air and in the region of the molding tool.

Introduction. Direct resistance heating of metals and alloys have been widely used in industry, in particular, in the manufacture of anchor heads for the reinforcement of building structures by heating in air, heating of powder materials in a container for their sintering to a specified level of porosity, etc. The process of resistance heating is easily automated and combined (for the contact method of heating) with the process of deformation of a billet. A progressive technology related to resistance heating is electric upsetting, which allows one to produce details of complex shape with high accuracy [1–3]. Electric upsetting is performed in two steps. Initially, the billet is heated by an alternating current of commercial frequency $f = 50$ Hz. After that, the current is disconnected and the billet is deformed to a specified configuration. A diagram of direct resistance heating of a billet for electric upsetting is shown in Fig. 1. Because of the short duration of the second stage of electric upsetting, the temperature field in the billet is formed at the stage of direct resistance heating [4].

At the second stage of electric upsetting, an important condition for implementation of the molding process is precise local heating only of the region of the billet that undergoes deformation, taking into account the plastic properties of the treated material. For the majority of materials, the maximum degree of deformation accumulated before failure is in a narrow range of temperature θ above the Curie temperature θ_C [5]. The temperature distribution and value in the heated region should be predetermined, taking into account the configuration of the finished article and the limitations due to the thermal strength of the tool. Only satisfaction of these conditions makes it possible to produce details of required shape without flash and bur and with a precisely filled die [2, 4].

The existing models describing the resistance heating stage ignore the coupling of the electromagnetic and temperature fields in the volume of bodies in contact, use the solution of the classical problems of penetration of a plane electromagnetic wave into a half-space or the problem of electromagnetic-field distribution in an infinite conducting cylinder, and, in many cases, explain the physical picture of heating only qualitatively [1, 6–8]. We note that the skin effect in ferromagnetic materials is significant even at low frequencies [9, 10]. Deformation of a billet without failure under the temperature–force limitations due to the strength of the tool requires knowledge not only of the volume-average temperature (the so-called "forging" temperature) but also the distributions of electromagnetic and temperature fields and the history of their formation. In the present work, we develop a mathematical model of direct resistance heating for a system of axisymmetric

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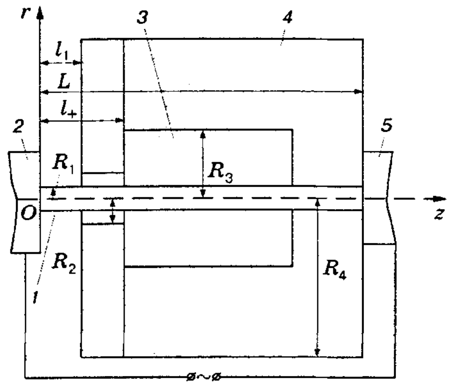

Fig. 1. Diagram of direct resistance heating of a billet for electric upsetting: billet (1), punches (2 and 5), removable die (3), and die holder (4): Oz is the symmetry axis, L is the length of the billet, l_+ is the length of the upset part of the billet, $l_1 = L - l_+$, and R_1, R_2, R_3, and R_4 are the radii of the billet, the upset part, the removable die, and the die holder, respectively.

ferro- and paramagnetic dissimilar bodies taking into account the skin effect and the temperature dependences of electromagnetic properties [electric conductivity σ (or resistivity $\rho = 1/\sigma$) and relative permeability μ_r] and thermal properties (thermal conductivity λ, specific heat c, and density γ) of materials. We note that the sharp peaks in the distribution of the specific heat c and the inflections of the parameters λ, γ, and ρ near the Curie point θ_C are due to a phase change with loss of magnetic properties in the neighborhood of this point ($\mu_r = 1$ at $\theta \geqslant \theta_C$). At $\theta < \theta_C$, μ_r practically does not depend on temperature and decreases rapidly to unity in a narrow temperature region containing the point θ_C [11].

1. Formulation of the Problem. We use Maxwell equations in a fixed reference system for high-conducting bodies ignoring bias currents and, hence, the dielectric properties of materials, which are preserved up to frequency $\omega = 2\pi f = 10^9$ sec^{-1}. In the region "billet–tool" for axisymmetric bodies, these equations can be reduced to the equation

$$\nabla \times \rho(\theta)\nabla \times \boldsymbol{H} = -\frac{\partial \boldsymbol{B}}{\partial t}, \qquad (1.1)$$

which, together with the relation for magnetically soft materials,

$$\boldsymbol{B} = \mu_0\mu_r(\theta, |\boldsymbol{H}|)\boldsymbol{H}, \qquad (1.2)$$

and the conditions of equality of the tangential components of the electromagnetic-field strength vectors in passage through the boundary between dissimilar media,

$$H_{1\tau} = H_{2\tau}, \qquad E_{1\tau} = E_{2\tau}, \qquad (1.3)$$

has a unique solution [12]. In (1.1)–(1.3), $\boldsymbol{H} = \boldsymbol{H}(r, \varphi, z, t)$ and $\boldsymbol{E} = \boldsymbol{E}(r, \varphi, z, t)$ are the magnetic- and electric-field strength vectors, respectively, $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of vacuum, $\boldsymbol{B}$ is the magnetic-induction vector, r, φ, and z are cylindrical coordinates, and t is time. Then, the density of heat sources q_v can be defined using Maxwell equations and Ohm's law:

$$\nabla \times \boldsymbol{H} = \boldsymbol{j}, \qquad \boldsymbol{j} = \sigma(\theta)\boldsymbol{E}. \qquad (1.4)$$

Under the Joule–Lenz law, $q_v = \rho(\theta)j^2$, where $\boldsymbol{j} = \boldsymbol{j}(r, \varphi, z, t)$ is the current-density vector.

The current-continuity equation as a consequence of the first of Eqs. (1.4) leads to the condition of equality of the normal components of the current density on the boundaries of dissimilar regions 1 and 2:

$$j_{1n} = j_{2n}. \qquad (1.5)$$

The air surrounding the region "billet–tool" is considered in a vacuum approximation, where $\boldsymbol{j} \equiv 0$. Obviously, on the boundaries of the contact "conductor–vacuum," relation (1.5) is converted to the conditions of "nonpenetration" of the current through the surface of the conductor:

$$j_n = 0. \tag{1.6}$$

The scalar and vector potentials of the electromagnetic field, which are frequently used to simplify the Maxwell equations, lead in this case to a coupled system of nonlinear differential equations for the vector-potential component. Therefore, we formulate the boundary-value problem of electrodynamics for the magnetic-field strength $\boldsymbol{H}$ [Eq. (1.1)] with relations (1.2), (1.3), (1.5), and (1.6).

Let $S = S_T \cup S_p$ be a region consisting of open regions of the tool S_T (die and die holder) and the billet S_p. The boundary $\Gamma = \Gamma_{\rm ext} \cup \Gamma_{\rm int}$ of the region $S^* = S \cup \Gamma$ includes the external boundary $\Gamma_{\rm ext} = \Gamma_v \cup \Gamma_h \cup \Gamma_a$, which consists of the boundary of contact of the billet with the punch (movable and fixed) Γ_h, the symmetry axis Γ_a (axis Oz), and the boundary of contact of the billet and the tool with air (in the vacuum approximation) Γ_v. The boundaries $\Gamma_{\rm int}$ are the internal boundaries of contact of the billet with the die and the die with the die holder for S. We consider the possibility of simplifying Eq. (1.1).

Writing the Maxwell equations in a fixed coordinate system (r, φ, z), we note that in the case of axial symmetry of the field, the strengths $\boldsymbol{H}$ and $\boldsymbol{E}$ are superpositions of the strengths of two fields:

$$\boldsymbol{H} = \{0, H_\varphi, 0\} + \{H_r, 0, H_z\}, \qquad \boldsymbol{E} = \{0, E_\varphi, 0\} + \{E_r, 0, E_z\},$$

and the magnetic field with the component H_φ determines values of E_r and E_z independently of the field with the component E_φ, which determines the components H_r and H_z [13].

We assume that the current I_0 is supplied at the ends of the billet parallel to the Oz axis and $\boldsymbol{n}$ is the outer unit normal vector to $\Gamma_{\rm ext}$. Then, for the field $\{H_r, 0, H_z\}$, the tangential component $\boldsymbol{n} \times \boldsymbol{H} = 0$ on Γ_v. Obviously, at the ends of the billet and on the symmetry axis, $H_r = 0$. Hence, $\boldsymbol{n} \times \boldsymbol{H} \equiv 0$ everywhere on $\Gamma_{\rm ext}$. Using the Poynting theorem in integral form, we can show that the electromagnetic-field energy decays in the region with the specified boundary conditions.

Thus, for the steady stage of resistance heating there is no field with the components $\{H_r, 0, H_z\}$ and $\{0, E_\varphi, 0\}$. Therefore, the electromagnetic field in the region S is determined by the transverse-magnetic field $\{0, H_\varphi, 0\}$ or an electric-type field.

Since in the region there is no heterogeneity along the azimuthal coordinate, we assume that the components of the electromagnetic-field strengths do not depend on φ. Therefore, for the region S, Eq. (1.1) can be written as a scalar equation for the component H_φ.

To determine the boundary conditions, we consider the first of Eqs. (1.4) outside the region S^* in the vacuum approximation, i.e., $\nabla \times \boldsymbol{H} = 0$. Then, H_φ does not depend on z and the magnetic field is defined by the relation

$$H_\varphi = c/r,$$

where c is a constant that is evaluated with allowance for (1.3), (1.5) and (1.6) from the Ampere law, which is an integral analog of Eq. (1.4), written for S^*.

Taking into account that $2\pi/\omega \ll T$ (T is the characteristic time of change in temperature in the system of bodies), we write the magnetic strength as

$$H_\varphi = H_\varphi(r, z, t) \exp(-i\omega t),$$

where $i^2 = -1$ and $H_\varphi(r, z, t)$ is the complex oscillation amplitude, which is slowly varying with time t. Similarly, the current I_0 and the magnetic induction B_φ are written as sinusoidal functions of time. This representation B_φ is reasonable since the higher harmonics that arise for the magnetic induction because of the nonlinearity of relation (1.2) are insignificant and, for steel, they account for not more than 5% of the first harmonic [9]. Then, without changing the designations and ignoring the term containing $\partial\mu_r/\partial t$ (it is different from zero only in the neighborhood of the Curie point [14]), we obtain a coupled boundary-value problem of thermoelectrodynamics in the region "billet–tool," which is written in operator form as follows.

For problem $M[H, \theta] = 0$,

$$\frac{\partial}{\partial r}\Big(\frac{\rho(\theta)}{r}\,\frac{\partial}{\partial r}\,(rH_\varphi)\Big) + \frac{\partial}{\partial z}\Big(\rho(\theta)\,\frac{\partial H_\varphi}{\partial z}\Big) + i\omega\mu_0\mu_r(\theta, H_\varphi)H_\varphi = 0 \quad \text{in} \quad S;$$

the boundary conditions are

$$H_\varphi = 0 \text{ for } r = 0, \quad H_\varphi = \frac{I_0}{2\pi a}, \quad a = \begin{cases} R_1, & r = R_1, \ 0 \leqslant z \leqslant l_+, \\ R_2, & r = R_2, \ l_1 \leqslant z \leqslant l_+, \\ R_4, & r = R_4, \ l_1 \leqslant z \leqslant L. \end{cases}$$

$$H_\varphi = \frac{I_0}{2\pi r} f(r), \quad f(r) = \begin{cases} r^2/R_1^2, & 0 \leqslant r \leqslant R_1, \quad z = 0 \text{ and } z = L, \\ 1, & R_1 \leqslant r \leqslant R_2, \quad z = l_+ \text{ and } R_2 \leqslant r \leqslant R_4, \ z = l_1, \\ 1, & R_1 \leqslant r \leqslant R_4, \quad z = L \end{cases}$$

on the boundary Γ_{ext} and

$$H_{1\varphi} = H_{2\varphi}, \qquad \rho_1(\theta)\,\frac{\partial H_{1\varphi}}{\partial \boldsymbol{n}} = \rho_2(\theta)\,\frac{\partial H_{2\varphi}}{\partial \boldsymbol{n}}$$

on the boundary Γ_{int}.

For problem $T[\theta, H] = 0$,

$$c(\theta)\gamma(\theta)\,\frac{\partial \theta}{\partial t} = \frac{1}{r}\,\frac{\partial}{\partial r}\Big(r\lambda(\theta)\,\frac{\partial \theta}{\partial r}\Big) + \frac{\partial}{\partial z}\Big(\lambda(\theta)\,\frac{\partial \theta}{\partial z}\Big) + q_v \quad \text{in} \quad S;$$

The initial conditions are $\theta = \theta(r, z, t)$ and $\theta(r, z, 0) = \theta_0(r, z)$. The boundary conditions are

$$\frac{\partial \theta}{\partial r} = 0, \quad r = 0; \qquad -\lambda(\theta)\,\frac{\partial \theta}{\partial \boldsymbol{n}} = \alpha_{\text{eff}}(\theta - \theta^*) - \zeta q_h \ \text{ on } \ \Gamma_v \cup \Gamma_h$$

on the boundary Γ_{ext} and

$$\theta_1 = \theta_2, \qquad \lambda_1(\theta)\,\frac{\partial \theta_1}{\partial \boldsymbol{n}} = \lambda_2(\theta)\,\frac{\partial \theta_2}{\partial \boldsymbol{n}}$$

on the boundary Γ_{int}. Here α_{eff} is the effective heat-removal coefficient that takes into account heat convection ($\zeta = 1$) on Γ_h and convective and radiative heat exchange ($\zeta = 0$) on Γ_v. θ^* is the ambient temperature, and q_h is a heat source that takes into account the transient contact resistance [3] on Γ_h.

The volume density of the heat sources is defined by

$$q_v = \frac{\rho(\theta)}{2}\Big(\Big|\frac{1}{r}\,\frac{\partial}{\partial r}\,(rH_\varphi)\Big|^2 + \Big|\frac{\partial H_\varphi}{\partial z}\Big|^2\Big).$$

We note that the evolution of the electromagnetic field is determined by the thermal conditions and the amplitude of the field depends on time as on a parameter.

Direct resistance heating can be implemented not only for a constant value of the current I_e but also for a constant value of the voltage U_e. Then, the boundary conditions for H_φ are nonstationary, i.e., for the magnetic-field strength, $I_0 = I(t)$, where $I(t)$ is the slowly varying amplitude of the current.

The problem of controlling the technological process of resistance heating can be solved using a simplified formulation of the problem in which the presence of a die is allowed for by heat exchange with the tool under Newton's law [15, 16].

2. Algorithms of Solution and Numerical Implementation. For joint solution of the problems on the introduced grid of values of time, calculations are conducted by an iterative scheme of the form

$$M[H, \theta^{(s-1)}] = 0, \qquad T[\theta, H^{(s)}] = 0 \qquad (s = 1, 2, \ldots). \tag{2.1}$$

In each time step, we calculate the approximation $H^{(s)}$ for the temperature $\theta^{(s-1)}$, and then solve the problem $T[\theta, H^{(s)}] = 0$, which determines the next approximation of the temperature field $\theta^{(s)}$. The iterative process (2.1) is interrupted when the required accuracy $\|\theta^{(s)} - \theta^{(s-1)}\| < \varepsilon$ is reached. This procedure divides the nonlinearly coupled problems in each step [17] and allows one to find the fields $\boldsymbol{H}$, $\boldsymbol{E}$, and θ by independent algorithms.

The formulated problem is characterized by discontinuities of the thermal and electrophysical properties on the boundaries of dissimilar regions. Therefore, the region S^* is divided by a nonuniform coupled grid along r and z. Roots of the Chebyshev polynomial are used as nodal values, and the boundary nodes belong to the boundary Γ_{ext} in the problem $M[H, \theta] = 0$. In the problem of determining the temperature field, the boundary Γ_{ext} is in the middle between the extreme nodes of the grid. In the problem of determining H_φ, the nodes belong to the boundary Γ_{int}, and in the problem $T[\theta, H] = 0$, this boundary is located in the middle between the nodes. Such discretization of the region provides for the second order of accuracy in approximating the boundary conditions of heat exchange, and on the internal boundaries, it does not require formulation of separate boundary conditions [18, 19].

Each of the indicated problems reduces to a system of algebraic equations by constructing a conservative difference scheme for the original differential problem [18, 19]. The time when the temperature reaches the Curie point is determined during solution.

The problem $M[H, \theta^{(s-1)}] = 0$ for a complex-valued equation of the type of the Helmholtz equation is solved in each time step by the Seidel method. The problem $T[\theta, H^{(s)}] = 0$ is solved by a locally one-dimensional method using a purely implicit, absolutely stable, difference scheme [18]. The relation $\mu_r(\theta, |\boldsymbol{H}|)$ is approximated by the universal magnetization curve using the basic values of μ_r and H_φ determined from experimental data for each of the materials [20].

All experimental data on the properties of the materials and calculated values of H_φ are interpolated by a rational spline of the third order, which ensures continuity up to the second derivative inclusively.

In each time step, calculations for $U_e = \text{const}$ using the Poynting theorem [8, 12] give values of the resistance r^* and the reactance x^*, which, besides being integral characteristics of the energy expended in heating and magnetic-field variation, are used to calculate the electric circuit of the resistance heating facility. Hence, from the expression $I = U_e/\sqrt{r^{*2} + x^{*2}}$, we can calculate the total current in the billet, which is iteratively calculated in the solution of the problem $M[H, \theta^{(s-1)}] = 0$ in this step. In calculations for $I_e = \text{const}$, the voltage depends on time and is defined by $U = I_e\sqrt{r^{*2} + x^{*2}}$. An additional iterative procedure in the last case is not required.

The test calculations performed indicate that the mathematical model is correct and appropriate for the algorithms used and that the adopted simplifying hypotheses are valid [15, 16].

3. Discussion of Results. The detected regularities of the formation of the temperature field in time and volume for direct resistance heating of billets of various standard sizes show that the temperature difference along the radius of the upset part is minor and the temperature on the billet surface here is higher than that on the axis up to the Curie point, after which the temperature on the axis is somewhat higher than on the surface. The temperature distribution along the length is more complicated (high gradients at the contact with the punch, in particular, at the entrance to the die). Figure 2 shows the temperature distribution at various times τ ($\tau = 10$ sec is the end of heating) in a 30Kh13 ferromagnetic steel billet 12.8 mm in diameter and 37 mm long at $I_e = 3600$ A.

The temperature distribution described above is typical of all standard sizes of billets, and it is practically impossible to produce a uniform temperature field of the upset part of the billet. This is explained by the considerable nonuniformity in the density of the heat sources due to the strongly nonuniform distribution of the current density j_z along the radius of the billet, which takes place up to the attainment of the Curie point in the upset region and during the entire heating process in the remaining region near the contact with the tool (Fig. 3). There is a characteristic "lateral displacement" of the current density, which was described for the first time in [6]. The component j_r is also distributed nonuniformly, but its value is an order of magnitude lower than that of j_z.

Generally, the mutual effect of the electromagnetic and temperature fields corresponds to the induction heating process described in [6–8]. The situation is, however, complicated by fact that the billet has two characteristic regions: the upset part and the part located in the die. In the part of the billet that is not upset, only the initial stage of formation of the electromagnetic and temperature fields is observed, which is "retarded" by intense heat exchange with the tool.

Figure 4 shows the distributions of the resistance and reactance over the heating time, which charac-

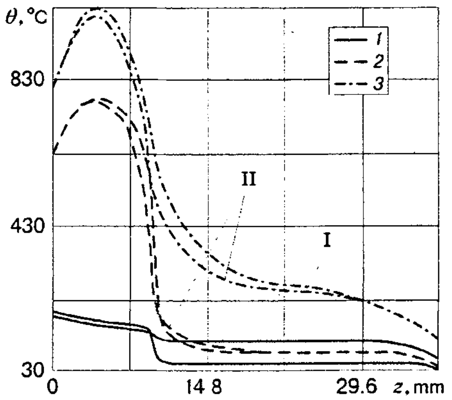

Fig. 2

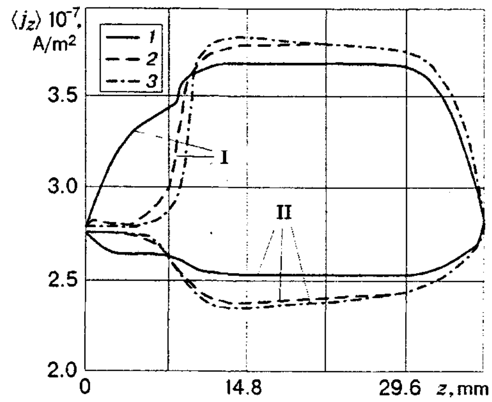

Fig. 3

Fig. 2. Temperature distribution along the length of the billet on the axis ($r = 0$) (curves I) and on the surface ($r = R_1$) (curves II) for $\tau = 1$ (1), 2 (2), and 10 sec (3).

Fig. 3. Distribution of the effective longitudinal component of the current density $\langle j_z \rangle$ along the length of the billet on the axis ($r = 0$) (curves I) and on the surface ($r = R_1$) (curves II) for $\tau = 1$ (1), 6 (2), and 10 sec (3).

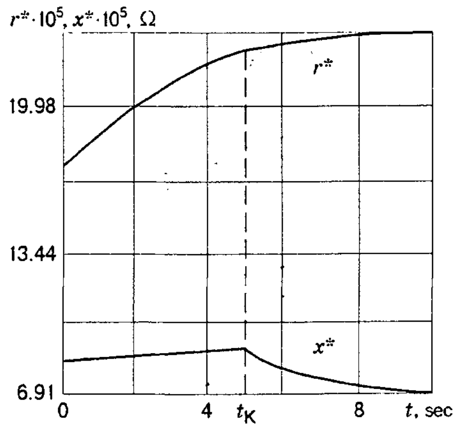

Fig. 4

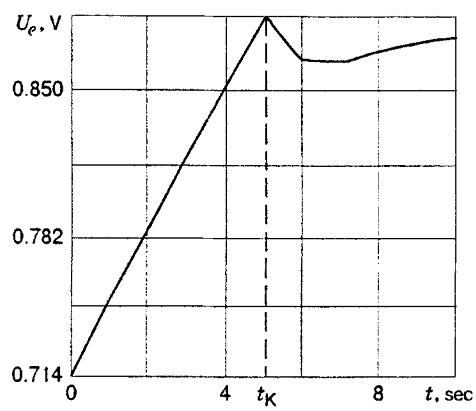

Fig. 5

Fig. 4. Distribution of the resistance (r^*) and reactance (x^*) of the billet over the heating time.

Fig. 5. Effective voltage across the billet versus heating time for $I_e = 3600$ A.

terize the power of heating of the billet and the magnetic field, respectively. The curves show that $r^* > x^*$ for all t and a fast increase in these parameters is observed up to the time $t = t_C$, which corresponds to the Curie point. After that, one observes inflection of the curves of $r^*(t)$ and $x^*(t)$ and a certain increase in the resistance due to an increase in $\rho(\theta)$. This behavior is typical of ferromagnetic materials and is more pronounced with strengthening of the surface effect beginning from $d \approx 16$ mm. For nonmagnetic materials, a monotonic increase in r^* and x^* is observed.

An important characteristic of the process is the voltage across the billet U, which varies with time in the heating regime $I_e = \text{const}$ and is determined from the obtained relations $r^*(t)$ and $x^*(t)$. The effective voltage across the billet U_e versus time is shown in Fig. 5. The characteristic point of inflection at the moment t_C corresponds to the Curie temperature of the surface layers of the upset part of the billet. At $t > 7$ sec, most of the upset part becomes nonmagnetic, the voltage drop ceases, and a small increase in U_e is observed (the increase in r^* is more considerable than the decrease in x^*).

Thus, knowledge of the important electrotechnical characteristics of the billet (resistances, reactance,

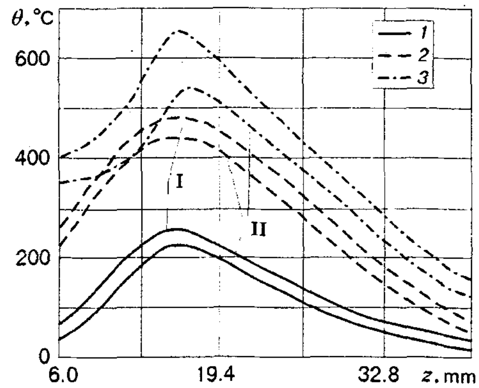

Fig. 6. Temperature distribution along the length of the die in the sections for $r = R_2 = 10.8$ mm (curves I) and $r = R_3 = 13$ mm (curves II); $\tau = 1$ (1), 6 (2), and 10 sec (3).

voltage, and their time dependences) gives necessary information for calculating the electric circuit of the direct resistance heating facility [2, 3].

In industrial production. the electric circuits of electric upsetting automatic control units are designed so that heating is performed in the regime U_e = const, which does not require expensive current regulators. Numerical solution of the problem of direct resistance heating is more conveniently performed for the regime I_e = const, since for the total time-dependent current in the billet, an additional iterative procedure is required in solving the problem $M[H, \theta] = 0$. This leads to additional computing costs, which are unacceptable in solving optimal control problems for resistance heating, where a calculation of a great number of versions is required. Therefore, we propose a procedure of conversion from the regime I_e = const to the regime U_e = const based on the equality of thermal energies in the heating interval from 0 to t_+ under the condition of invariance of the dependence $r^*(t)$, which ensures "recurrence" of the history of resistance heating. Then,

$$I_e^2 \int_0^{t_+} r^*(\tau)\, d\tau = U_e^2 \int_0^{t_+} \frac{d\tau}{r^*(\tau)}, \tag{3.1}$$

where the dependence $r^*(\tau)$ and the heating time t_+ are known from calculations for the regime I_e = const. The quantity U_e in expression (3.1) is corrected using the power coefficient of the facility [8], whose occurrence is due to the voltage component that overcomes the self-induction e.m.f. The calculations of direct resistance heating show the validity of this method of conversion [16].

Figure 6 shows the temperature-field distribution in two characteristic sections (see Fig. 1) of a hard-alloy die from VK-20 alloy. The die holder of radius $R_4 = 42.5$ mm was made of 4Kh4M2VFS tool steel. A billet of 30Kh13 steel with a radius of $R_1 = 6.5$ mm and length of 39 mm was heated. The Curie temperature is 980°C for VK-20 alloy and 820°C for 4Kh4M2VFS and 30Kh13 steels.

During the entire heating process, the temperature of the die remains higher than the temperature of the billet region that is not upset. The maximum temperature is near the outer surface of the die and is shifted to the depth of the tool. The temperature on the surface of the die holder does not exceed 80°C.

The current density and the heat sources are inversely proportional to the resistivities. On the surface of contact of the billet and the die. the power of heat sources varies jumpwise [21]. Since the resistivity of VK-20 alloy is about three times lower than the resistivity of the billet before it reaches the Curie point, the current "flows" into the die and passes basically through it and not through the undeformed part of the billet. The thermal activity $\sqrt{\lambda c \gamma}$ of VK-20 alloy is 30% higher than the thermal activity of the materials of the billet and the die holder. This leads to redistribution of the heat flows [21] from the region of contact of the billet and the die holder into the die. The temperature on the surface of the die holder is low because it is cooled with water.

Conclusions. Thus, the formulated nonstationary coupled initial-boundary-value problem of thermoelectrodynamics for the system of axisymmetric ferro- and paramagnetic dissimilar bodies taking into account the skin effect and the temperature dependences of both the electromagnetic and thermal properties of materials and the designed algorithms of solution make it possible to establish the regularities of formation of the electromagnetic and temperature fields in the region "billet–tool." This allows one to formulate and solve the problem of optimal control of this process (see, for example, [22]).

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